Q1. Consider $y_1, \ldots, y_n$ Gaussian random variables with mean μ, variance 1, and $cov(y_i, y_j) = \rho$, where $|\rho| < 1$. Denote $\bar{y} = n^{-1} \sum_{i=1}^{n} y_i$.

1. Show $\bar{y}$ is an unbiased estimator of μ.
2. Calculate the variance of $\bar{y}$.
3. If $\bar{y}$ an consistent estimate of μ?

Q2. The following plot shows locations of US weather stations.

Consider measuring air temperature, is the dataset collected a *geostatistial data, lattice data or spatial point process*? Please explain.

Q3. Use the Columbus dataset (For detailed information about datasets, type ?oldcol in R package "spdep"). Use Moran's I to detect spatial autocorrelation, and explain your findings. (e.g., choice of spatial weight matrix.)

Q4. Let $\boldsymbol{W} = [w_{ij}]$ be a spatial weight matrix and $\boldsymbol{u} = (u_1, \ldots, u_n)^\top$, where $u_i = Z(\boldsymbol{s}_i) - \bar{Z}$. Standardize the weights such that $\sum_{i,j} w_{ij} = n$. Let $\boldsymbol{Y} = \boldsymbol{W}\boldsymbol{u}$ and consider the regression through the origin $\boldsymbol{Y} = \beta\boldsymbol{u} + \boldsymbol{e}$, where $\boldsymbol{e} \sim (\boldsymbol{0}, \sigma^2\boldsymbol{I})$. What is measured by the slope β?

Q5. Let

$$Y(t) = Y(t-1) + e(t), \qquad t = 0, 1, 2, \ldots$$

where $Y(0) = 0$, $\{e(t)\}_{t=1,2,\ldots}$ is a sequence of independent random variables following $N(0, \sigma^2)$. Show that the process $Y(t)$ is not second-order stationary, but that the differences $D(t) = Y(t) - Y(t-1)$ are second-order stationary.